To-Do

Lecture 29 went into a bit more detail about λ -calculus. Note the following.

- 1. Continue working on HW-6. You should be about half way done with HW-6.
- 2. Study for Exam 2. Exam 2 is on Monday.
- 3. *Review the lecture notes.* Be sure to study the lecture notes and the examples.

Things to Know:

- You need to understand function applications in λ -calculus and how to do them.
- You need to understand how *true* and *false* can be represented in λ -calculus.
- You need to understand how to perform function applications to be able to evaluate basic boolean expressions in λ -calculus.
- Here are some practice exercises for you to do:
 - OR T F
 - OR F F
 - AND T T
 - AND T (NOT T)
 - NOT (AND T F)
 - OR (NOT (AND F F)) F
 - IF T THEN 1 ELSE 0
 - IF F THEN 1 ELSE 0

More on Recursion

In class it was mentioned that it is possible "stop" recursion using **IF**. Here is more detail on how this would work by trying to define a factorial function (recursively).

(a). First try at defining a factorial function FAC:

FAC
$$\equiv \lambda n.$$
 IF $(= n \ 1) \ 1 \ (* n \ (\underline{\text{FAC} \ (-n \ 1)}))$
undefined

(b). Instead, need to "parameterize" the self call ...

FAC $\equiv \lambda f.(\lambda n. \text{ IF } (= n \ 1) \ 1 \ (* \ n \ (f \ (-n \ 1))))$

(c). And apply the function using a ${\cal Y}$ combinator, for example:

$$(\mathbf{R} \text{ FAC}) \ 2 \equiv \text{FAC} (\mathbf{R} \text{ FAC}) \ 2 \\ \equiv (\lambda f.(\lambda n. \text{ IF } (= n \ 1) \ 1 \ (* \ n \ (f \ (-n \ 1))))) \ (\mathbf{R} \text{ FAC}) \ 2 \\ \equiv (\lambda n.(\text{IF } (= n \ 1) \ 1 \ (* \ n \ ((\mathbf{R} \text{ FAC}) \ (-n \ 1))))) \ 2 \\ \equiv \text{ IF } (= 2 \ 1) \ 1 \ (* \ 2 \ ((\mathbf{R} \text{ FAC}) \ (-2 \ 1)))) \\ \equiv (* \ 2 \ ((\mathbf{R} \text{ FAC}) \ 1)) \\ \equiv (* \ 2 \ (FAC \ (\mathbf{R} \text{ FAC}) \ 1)) \\ \equiv (* \ 2 \ (\lambda f.(\lambda n. \ \text{IF} \ (= n \ 1) \ 1 \ (* \ n \ (f \ (-n \ 1))))) \ (\mathbf{R} \text{ FAC}) \ 1)) \\ \equiv (* \ 2 \ (\lambda f.(\lambda n. \ \text{IF} \ (= n \ 1) \ 1 \ (* \ n \ (f \ (-n \ 1))))) \ (\mathbf{R} \text{ FAC}) \ 1)) \\ \equiv (* \ 2 \ (\lambda f.(\lambda n. \ \text{IF} \ (= n \ 1) \ 1 \ (* \ n \ (\mathbf{R} \text{ FAC}) \ (-n \ 1))))) \ 1)) \\ \equiv (* \ 2 \ (IF \ (= 1 \ 1) \ 1 \ (* \ 1 \ ((\mathbf{R} \text{ FAC}) \ (-1 \ 1))))) \ 1))$$

$$\equiv$$
 (* 2 1)