Lecture 16:
• Associativity and Precedence

Announcements:
• HW-3 out
• Quiz 4 on Friday (AST creation, visitors)
More on Context Free Grammars

With recursive descent parsers, it can be hard to ...

- define grammars with appropriate operator **associativity**
- define grammars with appropriate operator **precedence**
- ... and these are important for semantic analysis (and evaluation)

**Operator associativity**

- many operators are **left associative** ... e.g., $\times$, $\div$, $+$, $-$
- For example ... $40 \div 10 \div 2 \equiv (40 \div 10) \div 2$
- Can be captured by the grammar rule:

$$e \rightarrow e \div n$$

... $n$ a number

- and the “AST”:

```
  \div
 /   \
\div 2
```

```
40 10
```

- But notice this requires ulineleft recursion! ... so not $LL(k)$
Dealing with left-associative operators

- One approach is to **rewrite the AST** after parsing
  - similar to applying rotations in Red-Black or AVL trees

- Another is to **modify** the grammar and recursive-descent parser

- ... to construct the correct AST

**Example:**

The grammar rule (modified to be \( LL(k) \)) ...

\[
e \rightarrow n ( \text{DIVIDE} \ n )^*\]

- for **left-associative** ops use iteration (Kleene star) as above
- for **right-associative** ops use (tail) recursion (natural for recursive descent)

Modify the “normal” recursive descent function to build left-associative AST ...

```python
def e(self):
    expr_node = ValExpr(val=self.curr_token)
    self.eat(TokenType.INT_VAL, '...')
    while self.match(TokenType.DIVIDE):
        self.advance()
        sub_expr_node = ValExpr(val=self.curr_token)
        self.eat(Token.INT_VAL, '...')
        tmp = DivExpr(lhs=expr_node, rhs=sub_expr_node)
        expr_node = tmp
    return expr_node
```
Check In: Trace the code above and show the AST for $40 \div 10 \div 2$.

The result is:

```python
expr_node = ValExpr(40)
sub_expr_node = ValExpr(10),
expr_node = DivExpr(ValExpr(40), ValExpr(10))
sub_expr_node = ValExpr(2),
expr_node = DivExpr(DivExpr(ValExpr(40), ValExpr(10)), ValExpr(2))
```
Operator precedence

- Division (/) has higher precedence than addition (+)
- For example:
  \[
  2 + 3 / 4 \equiv 2 + (3 / 4) \\
  2 / 3 + 4 \equiv (2 / 3) + 4
  \]

One solution: Encode precedence in the grammar

\[
e \rightarrow t ( \text{PLUS } t )* \\
t \rightarrow \text{INT } ( \text{DIVIDE } \text{INT } )* \\
\]

- This is equivalent to ... 

\[
e \rightarrow t \ e' \\
e' \rightarrow \text{PLUS } t \ e' \ | \ \epsilon \\
t \rightarrow \text{INT } t' \\
t' \rightarrow \text{DIVIDE } \text{INT } t' \ | \ \epsilon \\
\]

Exercise: Draw the parse tree for: \(2 + 3 / 4 + 5\)

* Don’t need to consider associativity and precedence for HW-3
- but you should understand the issues and how to resolve them
- note it would be a good extension project
Summary – Things to Know

1. Difference between operator associativity and precedence.

2. The issue/challenge with encoding associativity into a grammar.

3. Options for “dealing with” associativity in a recursive descent parser.

4. Given an example, generate a recursive descent function that correctly builds a left-associative AST.

5. General approach for encoding precedence into a grammar.

6. Given an example, create a grammar that correctly encodes precedence.