Info-gap uncertainty in structural optimization via genetic algorithms

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Keywords: convex models, genetic algorithms, optimal design, trusses, uncertainty.

ABSTRACT: This paper focuses on info-gap uncertainty for structures optimized via genetic algorithms. Convex models, a non probabilistic method, are used to deal with uncertain static loads. They are particularly effective when an info-gap situation arises, i.e. in the presence of severe uncertainties. Specifically, the uniform bound convex model is employed. Through a superposition method, the structural response can be maximized to capture the worst-case scenario. Traditionally, the use of convex models requires the uncertain parameters to be bound within a convex set. Here, a series of nested convex sets is considered to allow the uncertainty to vary. Design curves are derived that represent a tradeoff between the desired uncertainty and the structural cost. These curves are useful tools in decision-making for the engineer.

1 INTRODUCTION

This paper focuses on info-gap uncertainty for optimal structures. Convex models (Ben-Haim and Elishakoff, 1990), a non probabilistic method to deal with uncertainties, are employed for static loads. The method requires that uncertain parameters be bound within a convex set. Convex models are able to capture the worst-case scenario due to uncertainties. They are very efficient when an info-gap situation arises, i.e. when the availability of data on an uncertain parameter is scarce. A superposition method (Ganzerli and Pantelides, 2000) is conveniently used to apply the uniform bound convex model and obtain the convex structural responses. The method is applicable to large structures with many members and uncertain parameters (Ganzerli and Burkhart 2002; Ganzerli et al., 2003). Convex models can be used in both conventional and optimal design of structures.

The necessity of setting bounds on the uncertainty is a weakness of the method. To counter this, one can create a nested series of convex sets: the longer the series and the larger the sets, the larger the uncertainty. Robustness is defined to be the greatest level of uncertainty at which failure cannot occur. Of course, it is advantageous to allow the uncertainties to vary considerably from the nominal value without the collapse of the structure (Ben-Haim, 1996). However, to tolerate a large uncertainty, it is necessary to sacrifice the performance of the design. One can think of the performance as the structural volume, expressed as a function of the cross-sectional areas and the load magnitudes. In optimal design terms, we would say that the performance is the value of the objective function dependent upon the design parameters and the uncertainties. Design curves can be developed to plot the robustness against uncertainties versus the structural performance. The design curves show the necessary tradeoff in decision-making between the structural cost, related to the volume, and the robustness. The design curves are a useful tool to the engineer. The decision of where to choose an optimal point on the design curve takes place during the design process, and it is dependent on the desired robustness for the structure.

This paper considers large trusses, with up to 64 independent design variables. Problems involving optimization of the structural volume, related to the cost, are the focus of the study. For safety and serviceability, constraints are imposed on member stresses and nodal displacements. A genetic algorithm (GA), coded by the authors of this paper, is used to carry out the optimal structural design.

In this section, the theory of convex models as applied to fractional uncertainty is introduced. Traditionally, the theory of convex models has been used in the optimal design of structures assuming that the loads are uncertain by a fraction of their nominal value. Later in this section, it is explained how the uncertainty can be allowed to vary without the need of confining it to set bounds. In other words, the robustness of the structure against several level of uncertainty is sought.

When considering fractional uncertainty, the convex model method requires the uncertain parameters to be bound within a convex set. Therefore, the loads vary from their nominal value of a fixed percentage of uncertainty β_n. Consider the truss of Fig. 1.

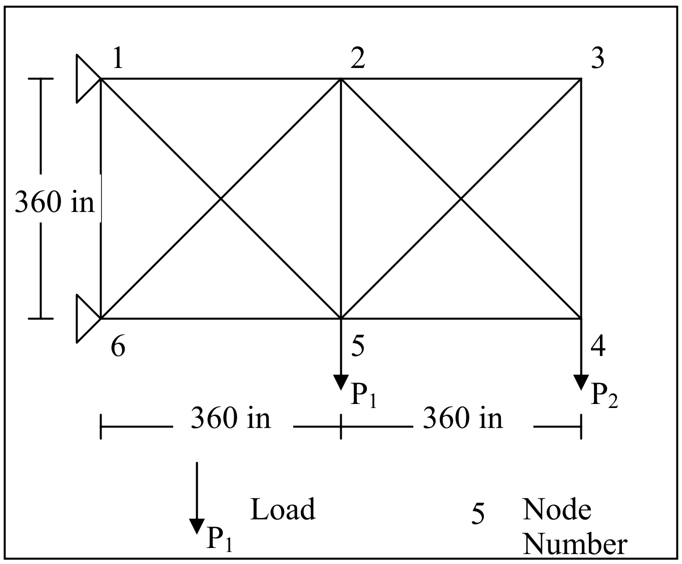

Figure 1. 10-bar truss.

Supposing that loads P_1 and P_2 are allowed to vary 10% from their nominal value, $\beta_1 = \beta_2 = 0.1$. This corresponds to the inner convex set represented in Fig. 2.

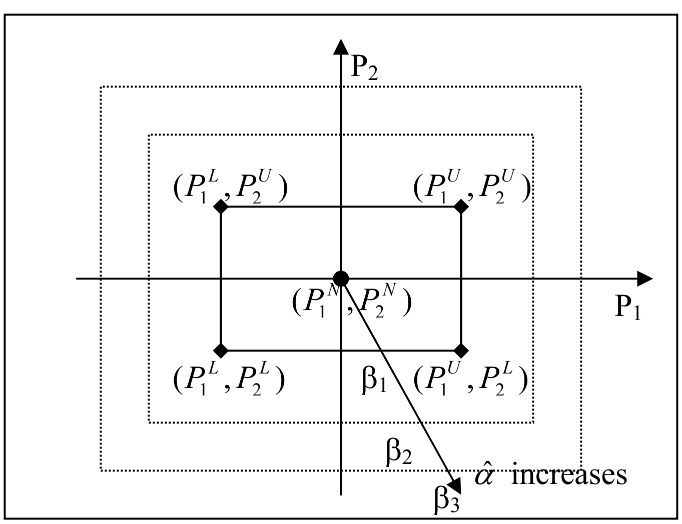

Figure 2. Nested convex sets.

Some definitions are necessary to understand Fig. 2. Hereafter, superscript N, U, and L will designate the terms nominal, upper, and lower respectively. The origin (P_1^N, P_2^N) is the point where no uncertainties are present and the loads assume their nominal values. The vertexes of the rectangle are the extreme variations of the loads from their nominal values. For example, the upper right corner with coordinates (P_1^U, P_2^U) is the point where the loads are both increased to their maximum. Upper and lower limits of the kth load can be related to the nominal values as follows:

$$P_k^U = P_k^N + \beta_n P_k^N \tag{1}$$
$$P_k^L = P_k^N - \beta_n P_k^N \tag{2}$$

Where

- k = number of loads

A numerical example, might clarify further how to build the convex set for fractional uncertainties. Let's say that (P_1^N, P_2^N)= (100 kip, 100 kip) and $\beta_1 = \beta_2 = 0.1$. The four vertexes of the convex set will have coordinates: (P_1^U, P_2^U) = (110, 110); (P_1^U, P_2^L) = (110, 90); (P_1^L, P_2^L) = (90, 90); (P_1^L, P_2^U) = (90, 110).

Once the convex set is defined, the convex models need to be implemented. To this end, a superposition method will be employed, that allows deriving equations for maximizing the structural response in terms of stresses and displacements. Here is how the method works. First, the structure is loaded with both P_1 and P_2 using the load nominal values. The nominal responses are obtained as x(P_1^N, P_2^N) for the nodal displacements and F(P_1^N, P_2^N) for the internal forces. Then the process is repeated similarly but using only one load at the time. In other words, the nominal structural response is obtained loading the structure with only P_1. x(P_1^N) and F(P_1^N) are calculated. The same is done with P_2 obtaining x(P_2^N) and F(P_2^N). Now, superposition can be applied to derive the convex structural response as follows:

$$\mathbf{x}_{\mathbf{i,\,con}} = \mathbf{x}_\mathbf{i}(P_1^N, P_2^N) \pm \{\beta_1\,|\mathbf{x}_\mathbf{i}(P_1^N)| + \beta_2\,|\mathbf{x}_\mathbf{i}(P_2^N)|\} \tag{3}$$

$$\mathbf{F}_{\mathbf{j,\,con}} = \mathbf{F}_\mathbf{j}(P_1^N, P_2^N) \pm \{\beta_1\,|\mathbf{F}_\mathbf{j}(P_1^N)| + \beta_2\,|\mathbf{F}_\mathbf{j}(P_2^N)|\} \tag{4}$$

$$\sigma_{\mathbf{j,\,con}} = \mathbf{F}_{\mathbf{j,\,con}}/A_j \tag{5}$$

Where

- i = number of degrees of freedom (varies from one to eight for the 10-bar truss)
- j = number of members (varies from one to ten for the 10-bar truss)
- $\mathbf{x}_{\mathbf{i,\,con}}$ and $\mathbf{F}_{\mathbf{j,\,con}}$ are the convex displacements and internal forces

- $\mathbf{x_i}(P_1^N, P_2^N)$ and $\mathbf{F_j}(P_1^N, P_2^N)$ are the nominal displacements and internal forces calculated loading the structure with both P_1^N and P_2^N
- $|\mathbf{x_i}(P_1^N)|$ and $|\mathbf{F_j}(P_1^N)|$ are the absolute values of the nominal displacements and internal forces calculated loading the structure with only P_1^N ($P_2 = 0$)
- $|\mathbf{x_i}(P_2^N)|$ and $|\mathbf{F_j}(P_2^N)|$ are the absolute values of the nominal displacements and internal forces calculated loading the structure with only P_2^N ($P_1 = 0$)
- β_1 and β_2 are the percents of uncertainty for P_1 and P_2 respectively.
- $\boldsymbol{\sigma}_{\mathbf{j,\,con}}$ are the convex stresses, that can be directly derived by the convex forces simply dividing them by the member cross-sectional areas (A_j).

In Eq. (3) and (4) the ± sign is in agreement with the sign of the first term. In other words, if $\mathbf{x_i}(P_1^N, P_2^N)$ and $\mathbf{F_j}(P_1^N, P_2^N)$ are positive the plus/minus sign will turn into a plus sign and vice versa. This guarantees that the nominal displacements are always increased when uncertainty is present and the worst-case scenario due to the uncertain parameters is captured by the equations.

Once the convex responses are calculated, they will be used in lieu of the nominal ones in the structural design process. This guarantees that through the structural response is maximized accounting for the uncertainty in the design parameters.

It is desirable to study different degrees of uncertainty. In order to do so, several values of β_n should be considered. Recalling that β_n is the percent of uncertainty, a nested series of convex sets is obtained. The larger β_n, the larger the set, the larger is the uncertainty. These convex sets are represented in Fig. 2. Consider that the level of uncertainty is measured by a parameter $\hat{\alpha}$. The robustness expresses the greatest level of uncertainty at which failure cannot occur. Therefore, it is advantageous to allow the uncertainties to have a large variation from the nominal value without the collapse of the structure. In other words, a large robustness is sought (Ben-Haim 2001). However, to tolerate a large uncertainty, it is necessary to sacrifice the performance of the design. The performance is the structural volume that depends upon the cross-sectional areas and the load magnitudes. For optimal structural design the performance coincides with the value of the objective function. The critical performance, r_c, is the minimum level of performance accepted.

The robustness can be plotted versus the performance to obtain a design curve, as shown in Fig. 3. It has been demonstrated that the design curve is monotonic (Ben-Haim 1996). This implies that there is a trade-off in deciding which point on the design curve is the "working point", i.e., the optimal design. The decision of where to choose the working point on the design curve takes place during the design process, and it is dependent on the desired robustness for the structure.

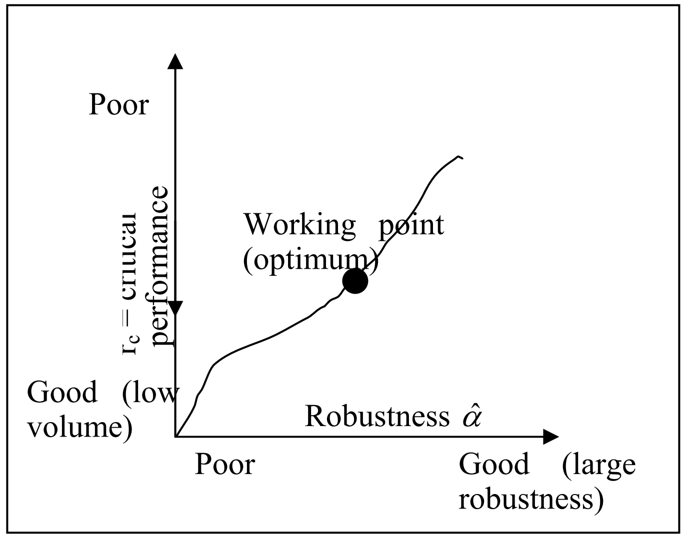

Figure 3. The design curve.

3 STRUCTURAL OPTIMIZATION

3.1 *Traditional optimization methods*

Traditional optimization techniques are calculus based (Kirsch, 1981). In conventional terms, the function to be optimized is called "objective function." In optimal truss design, the objective function is the volume that is related to the structural cost. The volume depends on the member cross-sectional areas A and the loads, P. These are called "design parameters." The loads here are also the uncertain parameters, keeping in mind that they are not deterministic. Structural optimization is subject to constraints. In order to satisfy safety and serviceability, stress and displacement constraints must be imposed. The structural optimization problem can be stated as:

$$\begin{aligned} &\text{minimize } V(A_j, P_k) \\ &\text{such that } x_i(A_j, P_k) \le x_{i,allowable} \\ &\qquad\qquad \sigma_j(A_j, P_k) \le \sigma_{j,allowable} \end{aligned} \tag{6}$$

Where

- As previously stated, i = number of degrees of freedom and j = number of members, k = number of loads
- V is the volume expressed as a function of the design parameters, i.e. the cross-sectional areas (A_j) and the external loads (P_k)
- $x_i(A_j, P_k)$ and $\sigma_j(A_j, P_k)$ are the constraints, i.e. the displacements and the stresses respectively

- $x_{i,allowable}$ and $\sigma_{j,allowable}$ are the allowable values for constraints $x_i(A_j, P_k)$ and $\sigma_j(A_j, P_k)$

In the traditional, calculus-based, optimization method the objective function needs to be continuous. This can create difficulties since not all functions are differentiable. Moreover, it is convenient at times limiting the search space to a discrete array. This is especially true in the structural engineering field, where sections often come in standard sizes. A newer approach, genetic algorithms, overcomes these difficulties.

3.2 *Genetic algorithms*

GA mimics the natural selection process (Haupt and Haupt, 1998). An initial "population" of design variable values is randomly selected and ranked. "Parents" that will mate and reproduce are the ones that possess the best characteristics, i.e. display a low volume and do not violate the set constraints. A second generation of "offspring" contains the "genes" of both parents. Some random mutation is introduced in the genes to avoid a quick convergence of the algorithm to a non-optimal value and to introduce values that were not included in the initial population. The second generation, composed of parents and offspring, is subjected to the same selective cycle as the previous generation. The process is continued until the population converges to the optimal design. The method is particularly efficient for discrete optimization. As just stated, this aspect is significant for structures, where the member cross-sectional areas are the design variables and are often available in standard sets.

4 EXAMPLES

This section presents two numerical examples. The minimal volume for trusses is sought where constraints on convex stresses and displacements from Eqs. (3-5) are imposed. The load condition is uncertain and, instead of imposing fixed boundaries, a range of uncertainties is considered. The aim is to derive design curves that would fit any level of uncertainty. To this end, for each truss, eleven structural designs were computed. First, the truss is optimized using the nominal loads. In this case there is no uncertainty and the parameters β_n of Eqs. (1-5) are equal t zero. The next ten designs are obtained increasing the uncertainties of increments of 10 percent. Graphing the volume versus the uncertainties, and plotting these eleven points, a design curve can be obtained. All the loads present on the structure are considered to be part of one load condition, and they are all increased of the same percentage. The two examples considered include a 10-bar truss and a 64-bar truss.

4.1 *10-bar truss*

The 10-bar truss represented in Fig. 1, is often used as a benchmark in the literature (Ganzerli and Burkhart, 2000). The truss is made of aluminum, and has a Young's modulus E equal to 10,000 ksi. Member stresses cannot exceed 25 ksi for both tension and compression. The only exception is member with end joints 2-4 that can reach a value of stress equal to ± 75 ksi. The nodal displacements are limited to 5 in.

4.2 *64-bar truss.*

The 64-bar truss represented in Fig. 4 has 28 nodes and 48 degrees of freedom. This truss is manufactured in aluminum and has a Young's modulus E equal to 10,000 Ksi. The length of all the horizontal and vertical members is 200 in. In this example, the allowable stresses for each member are set as ± 25 ksi (for both tension and compression). Constraints are also imposed such that the vertical displacement at node 1 and the horizontal displacement at node 9 are less than 10 in. The nominal values of the loads are given in Fig. 4. This truss was solved as an example of optimization using GA by Ghasemi et al. (1999). The example here presented has the same geometry and load condition of the one cited but two main peculiarities of the example here solved must be mentioned. In this paper, uncertainties are considered for the load condition, whether Ghasemi et al. (1999) have solved the nominal case. Furthermore, here 64 independent variables are considered. Ghasemi et al. (1999) have linked the 64 variables to obtain 19 groups of independent variables. Linking the variables is attractive for practical purposes. In structural design, it is more feasible to use a limited number of sections for ease of construction and economical benefits. However, the authors of this paper are using the 64 independent variables to demonstrate the power of GA in solving large problems. For sake of comparison, the authors of this paper, have solved the 64-bar truss as presented by Ghasemi et al. (1999) and obtained very close results. These are not presented here for brevity.

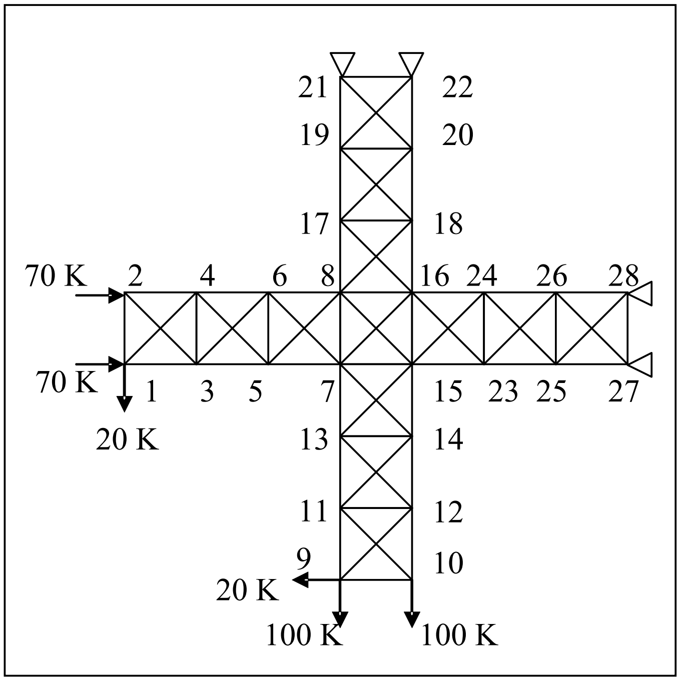

Figure 4. 64-bar truss.

4.3 *Results*

Table 1 includes the optimal volumes obtained for the 10-bar and 64-bar trusses. The results are presented for each percentile of uncertainty considered. This is varying from zero uncertainty (β_n = 0) to 100% uncertainty $\beta_n = 1$.

Table 1. Optimal results.

Fractional uncertainty β_n	10-bar truss optimal volume in^3	64-bar truss optimal volume in^3
0	15396	25789.6
0.1	16961.7	29365.9
0.2	18683.4	35367.2
0.3	20994.4	39088.6
0.4	22802.9	46197
0.5	24625.8	52668
0.6	25819.1	58142.9
0.7	27522.1	64114.2
0.8	29375	70986
0.9	31304.8	76779.1
1.0	33003	81549.7

Plotting the minimized volumes for the 10-bar truss found in Column 2 of Table 1 versus the uncertainties Figure 5 is obtained. The 11 points are connected with a line that is the design curve for the 10-bar truss. Similarly, plotting the volumes in Column 3 Table 1 versus the uncertainties and connecting the dots, the design curve for the 64-bar truss is obtained. See Fig. 6.

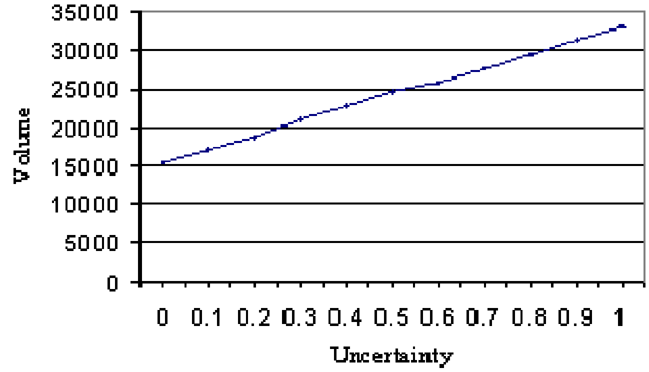

Figure 5. 10-bar truss. Design curve.

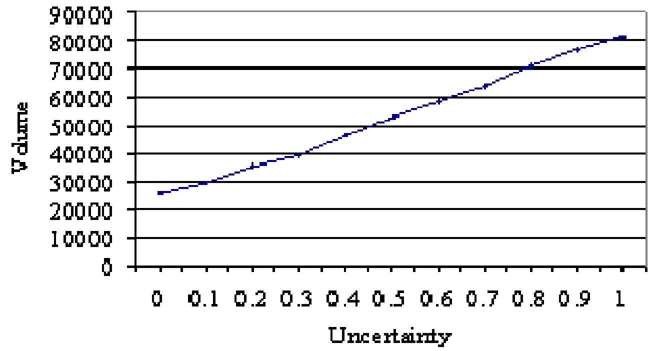

Figure 6. 64 bar truss. Design curve.

The design curves for the 10-bar truss and for the 64-bar truss are approximately straight lines. This means that the volumes grow in linear direct proportion with the load increase. The reason for this outcome is that the objective function, the structural volume, and the constraints, stresses and displacements, are linear functions of the design variables (the member cross-sectional areas). Deriving this result is important. In fact, the designer could calculate a limited number of points on the curve, maybe two or three. From these few points the all curve can be plotted for any level of uncertainty. The working point can then be determined based on considerations of safety, economy, and importance of the structure.

These curves are significant from a computational point of view. A criticism that optimization often receives, is that one can never be sure to have obtained the absolute optimum. Once a solution is reached the doubt remains that it is a local minima. Since these curves are straight lines, it is easy to check if a volume that is calculated for a certain percentage of uncertainty belongs to the curve or not. If the volume falls outside the curve, then the designer can investigate if a local minimum has been reached. Letting the algorithm run for more generations will produce more accurate results. Therefore, the design curves serve as a check to see if a structural design is a true minimum or not.

5 DISCUSSION AND CONCLUSION

In this paper, optimal structural design using GA was addressed. Convex models were used to address the uncertainties affecting static loads for trusses. Using the superposition method, design curves were derived that fit any level of uncertainty. Two important considerations need to be stated. The design curves are a plot of a series of optimal structural designs obtained augmenting the fractional uncertainties by equal increments, e.g. 0% nominal case, 10 % uncertainty, 20% uncertainty, and so on. The authors did not identify a working point on the curves with the intent of leaving this decision for the engineer to make. Recall from paragraph 2, that the working point is the optimum compromise between the cost and the tolerance to uncertainty that are inversely proportional. The curves are intended as design tools and the level of robustness against uncertainties sought is problem dependent. Think, for example, at the importance of a structure. A higher level of robustness is desirable for a public assembly place than for a warehouse.

The robustness of the structure is here intended as directly proportional to the uncertainty. In other words, the structural design problem was formulated assigning a level of uncertainty and calculating the minimum volume that the structure can have to satisfy that uncertainty. The coefficient α that measures robustness corresponds to the coefficients β_1. If we consider, for example, an uncertainty of 10 %, the reliability of the structure is the coefficient $\beta_1 = 0.1$. Another approach to robustness is to seek the performance of a design that is least sensitive to the variability of uncertain variables (Au et al., 2003). In the latter case, a fix volume is chosen maximizing the tolerance to the uncertain parameters. The coefficients β_n are not known a priori, but are calculated as objective functions during the optimization process. The authors of this paper have not chosen the second approach because, although a better structural performance could be obtained, this application is computationally expensive. Au et al. (2003) have used unsatisfactory functions to reach this goal. The method could be applied to derive design curves. However, using unsatisfactory functions requires performing a nested optimization. Even with using decomposition, that uncouples the nested optimization, the method is computationally expensive. The superposition method is attractive for its ease of implementation and guarantees the robustness against the uncertainty under consideration.

The authors are working towards developing design curves that allow different load conditions to be present on the structure simultaneously. The next step is to consider separate load conditions for which the uncertainty can vary independently from each other.

In conclusion, the superposition method has been used to derive design curves that fit any level of uncertainty. Convex models are an attractive alternative to the probabilistic method to deal with uncertainties. This paper explored the possibility of letting the uncertainties vary. This is a step forward with respect to the traditional convex model approach that dealt with fractional uncertainty.

6 APPENDIX

A Table for conversion factors from US to SI units is given below for the units used in this paper.

Table 2. Conversion factors.

inches (in)	millimeters (mm)	25.4
feet (ft)	meters (m)	0.305
pound force[a] (lb)	Newtons	4.448
Pounds per sq in[b] (psi)	Newtons per sq m (N/m^2)	6895

[a]Kips equal kilo-pounds = 1,000 lb
[b]Ksi equal kilo-pounds per square in = 1,000 psi

ACKNOWLEDGEMENTS

Funding for this project has been provided by the McDonald Work Award. Mr. and Mrs. McDonald have generously established this grant to the benefit of undergraduate students at Gonzaga University.

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