Tower of Hanoi

Iterative solution

The following solution is a simple solution for the toy puzzle. Alternate moves between the smallest piece and a non-smallest piece. When moving the smallest piece, always move it to the next position in the same direction (to the right if the starting number of pieces is even, to the left if the starting number of pieces is odd). If there is no tower position in the chosen direction, move the piece to the opposite end, but then continue to move in the correct direction. For example, if you started with three pieces, you would move the smallest piece to the opposite end, then continue in the left direction after that. When the turn is to move the non-smallest piece, there is only one legal move. Doing this will complete the puzzle using the fewest number of moves to do so.\(^5\)

It should perhaps be noted that this can be rewritten as a strikingly elegant set of rules:

Simpler statement of iterative solution

Alternating between the smallest and the next-smallest disks, follow the steps for the appropriate case:

For an even number of disks:

- make the legal move between pegs A and B
- make the legal move between pegs A and C
- make the legal move between pegs B and C
- repeat until complete

For an odd number of disks:

- make the legal move between pegs A and C
- make the legal move between pegs A and B
- make the legal move between pegs B and C
- repeat until complete

In each case, a total of \(2^n - 1\) moves are made

Equivalent iterative solution

Another way to generate the unique optimal iterative solution:

Number the disks 1 through \(n\) (largest to smallest).

- If \(n\) is odd, the first move is from the Start to the Finish peg.
- If \(n\) is even the first move is from the Start to the Using peg.

Now, add these constraints:

- No odd disk may be placed directly on an odd disk.
- No even disk may be placed directly on an even disk.
- Never undo your previous move (that is, do not move a disk back to its immediate last peg).
Considering those constraints after the first move, there is only one legal move at every subsequent turn. The sequence of these unique moves is an optimal solution to the problem equivalent to the iterative solution described above.

---