Instructions. Questions 1–4 ask you to generate proof trees and provide answers to the given queries. Questions 5–10 ask you to implement a set of relations in a file `hw12.pl`. Turn in both with a cover sheet and design document (for 5–10) as well as tests showing your relations work correctly in class by the due date. In your design document, discuss any challenges and/or issues you had in and assumptions you made. Also describe your strategy for testing your rules. Finally, discuss what happens when you use different “binding” patterns in the relations for questions 5–10.

1. Draw a proof tree and give the answers for the query `r(2)` given the following.
   
   ```
   p(1).
p(2).
q(2).
r(X) :- p(X), q(X).
   ```

2. Draw a proof tree and give the answers for the query `q(a,Y)` given the following.
   
   ```
   p(a,b).
p(b,c).
q(b,d).
q(X,Y) :- p(X,Z), p(Z,Y).
   ```

3. Draw a proof tree and give the answers for the query `p(X,Y,Z)` given the following.

   ```
   a(1,2).
b(2,3).
c(2,4).
p(X,Y,Z) :- a(X,Y), b(Y,Z).
p(X,Y,Z) :- a(X,Y), c(Y,Z).
   ```

4. Draw a proof tree and give the answers for the query `tc(a,Y)` given the following.

   ```
   e(a,b).
e(b,c).
tc(X,Y) :- e(X,Y).
tc(X,Y) :- e(X,Z), tc(Z,Y).
   ```

5. Write a rule(s) for a `distance((X1,Y1),(X2,Y2),D)` that computes the distance `D` between the two points `(X1,Y1)` and `(X2,Y2)`. For example, `distance((0,0),(2,0),D)` should result in `D` being bound to `2`.

6. Write a rule(s) for a `midpoint((X1,Y1),(X2,Y2),(X,Y))` relation that computes the midpoint `(X,Y)` between the two points `(X1,Y1)` and `(X2,Y2)`. For example, `midpoint((0,0),(2,0),(M))` should result in `M` being bound to `(1,0)`. 
7. Write a rule(s) for a $\text{median3}(X,Y,Z,M)$ relation that gives the median $M$ of the three value $X$, $Y$, and $Z$. For example $\text{median3}(5,2,3,M)$ should result in $M$ being bound to $3$. If there are not three unique values, return the value that occurs most frequently.

8. Write a rule(s) for a $\text{earlier_date}((M1,D1,Y1),(M2,D2,Y2))$ relation that is true if $(M1,D1,Y1)$ is an earlier date than $(M2,D2,Y2)$.

9. Write a rule(s) for a $\text{path}(X,Y,N)$ relation that computes all paths between nodes $X$ and $Y$ in a directed graph of length $N$. Assume a set of directed edge $\text{arc}(U,V)$ facts are given to represent the directed graph. For instance, if $\text{arc}(a,b)$ is true, implying there is an arc defined between a node $a$ and $b$, then $\text{path}(a,b,N)$ should result in $N$ being bound to $1$. Similarly, if $\text{arc}(a,b)$ and $\text{arc}(b,c)$, then $\text{path}(a,c,N)$ should result in $N$ being bound to $2$.

10. Write a rule(s) for a $\text{shortest_path}(X,Y,N)$ relation that returns the length $N$ of the shortest path in a directed graph between $X$ and $Y$. Your $\text{shortest_path}$ relation should be defined in terms of your $\text{path}$ relation.

Extra Credit. Write a rule(s) for an $\text{is_tree}(\text{Root})$ relation that returns true if the node $\text{Root}$ is the root of a tree within a (potentially) larger directed graph structured. Only consider nodes reachable from $\text{Root}$ for determining if $\text{Root}$ forms a tree structure. Let the an arc relation $\text{arc}(a,b)$ mean that the node $b$ is a child of $a$ when viewing the graph as a tree. Hint: Use your $\text{path}$ relation and use the tree constraint that a child can have at most one parent.